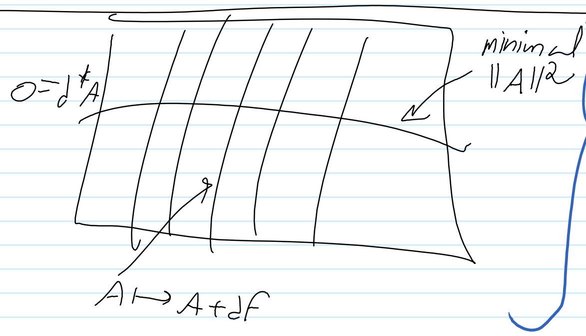
Friday-6 AKT on 140214: Gauge fixing in the Abelian case, perturbation theory in ${\mathbb R}^n$

February-12-14 8:33 AM

No HW meeting this evening, no new HW!
Enjoy the break! See web:DBN/Talks/Vienna-1402.

"Gauge Fixing"

$$Z(\Upsilon_R) = \int \mathcal{D}A \, e^{\frac{i}{4\pi}\int_{\mathbb{R}^3} A\wedge dA} \int_{\gamma_1} A \int_{\gamma_2} A$$



on board

What's d^*? "div", $*d*$

$$\delta(x) = \int e^{ixy}\,dy \qquad \delta(d^*A) = \int \mathcal{D}\phi\, e^{i2\int \phi\, d^*A} \qquad \phi\in\Omega^3$$

$$Z \mapsto \int \mathcal{D}A\,\mathcal{D}\phi\; e^{i\int A\wedge dA + 2\phi\, d^*A}$$

$$\int A\wedge dA + 2\phi\, d^*A = \left\langle \begin{pmatrix} A \\ \phi \end{pmatrix}, \underbrace{\begin{pmatrix} *d & d* \\ d* & 0 \end{pmatrix}}_{L^-} \begin{pmatrix} A \\ \phi \end{pmatrix} \right\rangle$$

in $\Omega^1\times\Omega^3$

$$(L^-)^2 = \pm\Delta \qquad (L^-)^{-1} = L^-(\Delta^{-1}) = \ldots$$

$$(\Delta^{-1}F)(x) = \int K(x,y)F(y)\,dy \qquad K(x,y) = K(\|x-y\|)$$

In $\mathbb{R}^n$, Δ^{-1} has units m^{-2} so K must have units $m^{2-n} \to m^{-1}$ in $\mathbb{R}^3$

$$\Rightarrow K(x,y) = C\frac{1}{\|x-y\|} \qquad C = \frac{1}{4\pi}$$

Knots and Feynman Diagrams, Jan 7 2002:

Emergence of FEYNMAN DIAGRAMS

Recall: we wish to understand $\int_{\{\text{conn.}\}} \mathcal{D}A\, e^{\frac{ik}{4\pi}\int_{\mathbb{R}^3} tr(A\wedge dA+\frac{2}{3}A\wedge A\wedge A)}\, hol_\gamma(A)$

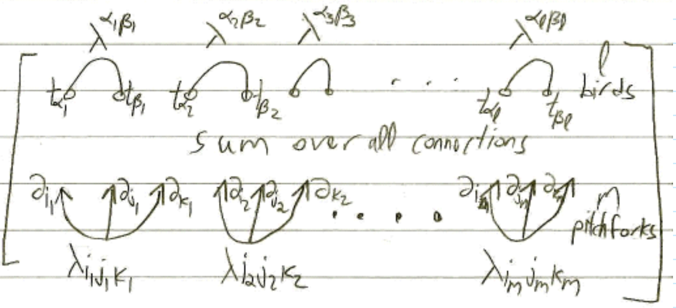
(whatever that may mean). As a warmup:

$$\int_{\mathbb{R}^n} dx\, e^{-\frac{1}{2}\lambda_{ij}x^ix^j+\frac{t}{6}\lambda_{ijk}x^ix^jx^k} = \frac{(2\pi)^{n/2}}{\det(\lambda_{ij})}\, e^{t\lambda_{ijk}\partial_i\partial_j\partial_k}\, e^{\frac{1}{2}\lambda^{\alpha\beta}t_\alpha t_\beta}\Big|_{t_\alpha=0}$$

$\partial_i := \frac{\partial}{\partial t_i}$

$$=\frac{(2\pi)^{n/2}}{\det(\lambda_{ij})}\sum_{\substack{m=0\\3m=2l}}^{\infty}\frac{1}{m!6^m l!2^l}\left(\lambda_{ijk}\partial_i\partial_j\partial_k\right)^m\left(\lambda^{\alpha\beta}t_\alpha t_\beta\right)^l$$

$$=\frac{(2\pi)^{n/2}}{\det(\lambda_{ij})}\sum_{\substack{m=0\\3m=2l}}^{\infty}\frac{1}{m!6^m l!2^l}$$



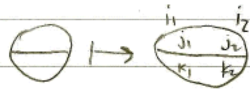
done

need repeating

$$=\frac{(2\pi)^{n/2}}{\det(\lambda_{ij})}\sum_{m=0}^{\infty}\frac{1}{m!6^m l!2^m}\sum_{\substack{m\text{-vertex fully marked}\\\text{Feynman Diagrams } D}}\mathcal{E}(D)$$

$D=$  , ...
but fully marked

$$=\frac{(2\pi)^{n/2}}{\det(\lambda_{ij})}\sum_{\substack{\text{unmarked}\\\text{Feynman}\\\text{Diagrams}\\D}}\frac{1}{Aut(D)}\mathcal{E}(D)$$

with $\mathcal{E}$:  $\longrightarrow \sum_{\substack{i_1j_1k_1\\i_2j_2k_2}}\left(\lambda_{i_1j_1k_1}\cdot\lambda_{i_2j_2k_2}\cdot\lambda^{i_1i_2}\cdot\lambda^{j_1j_2}\cdot\lambda^{k_1k_2}\right)$

Dror Bar-Natan

How many pairings will give a certain specific unmarked diagram?

A given one marking, $S_l\times(S_2)^l\times S_m\times(S_3)^m$ acts on the set of markings, but within that, $Aut(D)$ acts trivially. So the ans. is $\dfrac{l!2^l m!6^m}{|Aut(D)|}$